Roll No. | | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech.(ME) (2011 Onwards) (Sem.–5)

MATHEMATICS-III

Subject Code : BTAM-500

M.Code : 70601

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt ANY FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt ANY TWO questions.**

SECTION-A

1. Write briefly :

a) Expand $x - x^2$ in half range series in interval $(0, \pi)$ upto first three terms.

b) Find Laplace transform of $t^3 e^{-3t}$

c) Find the inverse Laplace Transform of $\left(\frac{4s + 15}{6s^2 + 25}\right)$

d) Describe the conditions required for the Fourier expansion.

e) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

f) Evaluate $\int x^3 j_0(x) dx$

g) Form the partial differential equation $z = (x + y) + (x^2 - y^2)$

h) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

i) Define harmonic function

j) Check whether $f(z) = \sqrt{|xy|}$ is analytic at origin or not?

SECTION-B

2. Obtain Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2,\ 0 < x < 2\pi$

3. Solve the initial value problem

$$y'' - 5y' + 4y = e^{2t},\ y(0) = \frac{19}{12},\ y'(0) = \frac{8}{3}$$

4. Find the frobenius series solution about $x = 0$ of equation

$$(1 - x^2)\,y'' - 2xy' + 6y = 0$$

5. Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Hence find

   a) The image of $|z| < 1$

   b) Invariant points of transformation.

6. Solve $(x^2 - yz)p + (y^2 - zx)\,q = z^2 - xy$

SECTION-C

7. State and prove convolution theorem. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$

8. Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles and hence evaluate $\int_c f(z)\,dz$ where c is circle $|z| = 2.5$

9. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions

$$u(0, y) = u(x, y) = u(x, 0) = 0,\ u(x, a) = \frac{\sin n\pi}{l}$$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.